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THE TRANSPORT FROM A DROP IN AN ALTERNATING ELECTRIC FIELD*

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Abstract—Analyses of heat and mass transfer from a drop in an electric field have, to date, dealt only with steady electric fields. This study presents both high and low Péclet number solutions for transport in an alternating electric field. The low Péclet number transport is investigated analytically using a composite, double perturbation expansion. Special analytic methods are developed to consider the previously untreated transport problem where the steady and time-dependent components of the fluid motion are of equal magnitude. A digital computer is employed to obtain exact solutions to the recursive governing equations. These solutions yield accurate results for Péclet number in the range 0–30 and thermal vibration number above 200.

NOMENCLATURE

a,	drop radius;
A_n,	constant, equation (60);
B_n,	constant, equation (61);
C_p,	$(ip)^{1/2}$;
D,	constant;
E,	magnitude of applied field;
F,	arbitrary function of R;
G,	solution, equations (64), (65) and (67);
H,	homogeneous solution;
i,	$\sqrt{-1}$;
Nu,	Nusselt number;
Pe,	Péclet number, $2Ua/\alpha$;
r,	spherical radial position;
R,	dimensionless radial position, r/a;
t,	time;
T,	local temperature;
$\mathbf{U}$,	fluid velocity;
$\mathbf{V}$,	dimensionless fluid velocity, $\mathbf{U}/U_0$;
x,	inner variable, $(R-1)/\delta$;
y,	$(\sqrt{2})(1+i)R/2\delta$;
Z,	$\cos(\theta)$.

Greek symbols

α,	thermal diffusivity;
β,	χ_1/χ_2;
δ,	$1/(\zeta^{1/2})$;
ε,	one-half Péclet number, U_0a/α;
ε_0,	permittivity of free space;
ζ,	thermal vibration number, $2\omega a^2/\alpha$;
η,	coefficient of fluid speed;
θ,	spherical polar angle;
κ,	dielectric constant;
λ,	κ_1/κ_2;
μ,	fluid viscosity;
τ,	dimensionless time, $2\omega t+\phi$;
T,	dimensionless temperature, $(T-T_\infty)/(T_a-T_\infty)$;
ϕ,	phase angle;
χ,	resistivity;
ω,	angular frequency of applied field.

Subscripts

1,	drop;
2,	surrounding fluid;
∞,	far from drop ($R\to\infty$);
a,	drop surface ($R=1$);
m, p, n,	degree of solution, equation (36);
r, R,	radial component;
s,	steady part;
t,	time dependent part;
z, θ,	tangential component.

Superscripts

j,	order of solution in ε;
k,	order of solution in δ.

INTRODUCTION

WHEN a uniform electric field is applied to a dielectric fluid in which a drop of another dielectric fluid is suspended, a charge build-up occurs on the drop–surrounding-phase interface. The field acts on the charge distribution to produce stress on the drop surface. Tangential components of the electric stress are balanced by fluid motion within and outside the drop. Such motion is of practical interest as it will enhance the direct contact heat or mass transfer rate between two immiscible fluids. Experimental studies by

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Thornton [1], Kozhukhar and Bologa [2], and Kaji *et al.* [3, 4] have shown remarkable increases in heat and mass transfer rates through use of electric fields.

The low Reynolds number creeping flow generated within and outside a single drop in a steady, uniform electric field has been analyzed by Taylor [5]. The drop is assumed to remain spherical and the convection of surface charge by fluid motion is neglected. Taylor found that either of two directions of fluid motion may be produced and that the direction is independent of the orientation of the applied field. In addition, that study provides a first-order approximation to field induced deformation of the drop. An extensive theoretical and experimental investigation of drop deformation has been performed by Allan and Mason [6].

Based on Taylor's conclusions, Griffiths and Morrison [7] analyzed the steady, low Péclet number transport in a steady electric field. The transient, high Péclet number transport has been analyzed by Morrison [8]. He found that the heat or mass transfer rate is proportional to the magnitude of the applied field. Both studies showed that the overall transfer rate is independent of the direction of fluid motion.

Taylor's results have been extended by Stewart and Morrison [9] to include small Reynolds number effects on the flow field. Sozou [10, 11] and Torza *et al.* [12] have, in addition, considered several time dependent problems associated with the fluid motion generated by both steady and alternating electric fields.

The purpose of this study is to examine the quasi-steady heat and mass transfer from a drop in an alternating electric field. The analysis is limited to the low Reynolds number or creeping flow domain. However, because dielectric fluids possess high Prandtl or Schmidt numbers, the entire range of Péclet number is physically meaningful.

GOVERNING EQUATIONS

The quasi-steady creeping flow about a drop in an alternating electric field was analyzed by Torza *et al.* [12]. They considered an oscillating field of the form

$$E = E_0 \cos(\omega t) \tag{1}$$

ω is the angular frequency and t is time. Their analysis neglects both local and convective acceleration effects on the fluid motion. This is valid when the Reynolds number and fluid vibration number are negligibly small. To first order, they found that the radial and tangential components of the fluid velocity outside a spherical drop of radius a are

$$\mathbf{U}|_r = U[(a/r)^2 - (a/r)^4](3Z^2 - 1) \tag{2}$$

and

$$\mathbf{U}|_\theta = -2U(a/r)^4 Z(1-Z^2)^{1/2} \tag{3}$$

where $Z = \cos(\theta)$. As shown in Fig. 1, r is the distance from the drop center and θ is the spherical polar angle. The maximum fluid speed U is given as

$$U = U_s + U_t \cos(2\omega t + \phi). \tag{4}$$

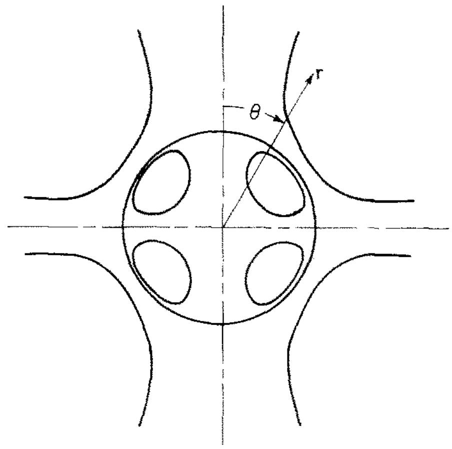

FIG. 1. Streamlines of the electrically driven circulating flow.

The subscripts s and t refer to steady and time-dependent parts of the fluid speed. ϕ is a phase angle and is dependent on the angular frequency of the field and properties of the drop and surrounding fluids. The steady part of the maximum fluid speed is

$$U_s = U_0 \eta_s \tag{5}$$

where

$$U_0 = \frac{9E_0^2 \varepsilon_0 \kappa_2 a}{10(\mu_1 + \mu_2)} \frac{\beta(\lambda\beta - 1)}{(2\beta + 1)^2} \tag{6}$$

and

$$\eta_s = \frac{1}{2}\left\{\frac{(2\beta+1)^2}{(2\beta+1)^2 + [\omega\varepsilon_0\chi_1\kappa_2(\lambda+2)]^2}\right\} \leqslant \frac{1}{2} \tag{7}$$

ε_0 is the permittivity of free space. κ, μ, and χ are the dielectric constant, viscosity, and resistivity of the fluids. The subscript 1 refers to the drop; the subscript 2 refers to the surrounding fluid. λ and β are the ratios of the dielectric constant and resistivity of the fluid in the drop to that outside.

Similarly, the time-dependent part is

$$U_t = U_0 \eta_t \tag{8}$$

where

$$\eta_t = \eta_s [1 + (\omega\varepsilon_0\kappa_2\chi_2)^2]^{1/2} \leqslant \tfrac{1}{2}. \tag{9}$$

Thus, U_0 represents the value of the steady plus the time dependent component of the fluid speed as ω tends to zero. In this limit, these results reduce to those found by Taylor [5] in his analysis of a drop in a steady electric field. As ω increases without bound, both components of the fluid speed approach zero. U_s falls off as $1/\omega^2$, while U_t goes as $1/\omega$. It is important to realize that this is not due to inertial effects in the flow field. Rather, it is due to the decrease in local charge build-up and the concomitant decrease in electric stress on the drop surface.

U_0 may be either positive or negative. When the product $\lambda\beta$ is less than unity, surface fluid motion is from the poles toward the equatorial plane. When it is greater than unity, the direction is reversed.

The quasi-steady temperature field is governed by the energy equation.

$$\alpha \nabla^2 T = \mathbf{U}\cdot\nabla T + \frac{\partial T}{\partial t}. \tag{10}$$

T is the local temperature and α is the thermal diffusivity. For a drop at uniform constant temperature T_a, in a fluid at uniform constant temperature T_∞ far from the drop

$$T = T_a \quad \text{on} \quad r = a \tag{11}$$

and

$$T \to T_\infty \quad \text{as} \quad r \to \infty \tag{12}$$

for all time.

The governing equations for mass transfer are analogous to those for heat transfer and need not be written separately. The corresponding expressions for mass transfer are obtained by replacing temperature by concentration and thermal diffusivity by molecular diffusivity.

We take as a reference length and reference speed, the drop radius a and the fluid speed U, respectively. The dimensionless radial position and fluid velocity are then

$$R = r/a \tag{13}$$

and

$$\mathbf{V} = \mathbf{U}/U. \tag{14}$$

The normalized temperature and time are

$$\mathrm{T} = (T - T_\infty)/(T_a - T_\infty) \tag{15}$$

and

$$\tau = 2\omega t + \phi. \tag{16}$$

Under these transformations, the energy equation (10) may be written

$$\frac{\partial \mathrm{T}}{\partial \tau} = \delta^2 \left\{ R^{-2}\frac{\partial}{\partial R}\left[R^2\frac{\partial \mathrm{T}}{\partial R}\right] + R^{-2}\frac{\partial}{\partial Z}\left[(1-Z^2)^{1/2}\frac{\partial \mathrm{T}}{\partial Z}\right] - \varepsilon[\eta_s + \eta_t \cos(\tau)] \times \left[\mathbf{V}|_R \frac{\partial \mathrm{T}}{\partial R} - R^{-1}\mathbf{V}|_\theta (1-Z^2)^{1/2}\frac{\partial \mathrm{T}}{\partial Z}\right]\right\}. \tag{17}$$

δ^2 is the inverse of the thermal vibration number, ζ.

$$\delta^2 = (2\omega a^2/\alpha)^{-1} = \zeta^{-1}. \tag{18}$$

The thermal vibration number is the ratio of the thermal relaxation time (a^2/α) to a characteristic time of the applied field $(1/2\omega)$. The parameter ε, one-half the Péclet number based on U_0, is introduced for convenience,

$$\varepsilon = U_0 a/\alpha = Pe/2. \tag{19}$$

In dimensionless form, the boundary conditions are

$$\mathrm{T} = 1 \quad \text{on} \quad R = 1, \tag{20}$$

$$\mathrm{T} \to 0 \quad \text{as} \quad R \to \infty. \tag{21}$$

ANALYSIS

An exact solution to the energy equation for arbitrary Péclet and thermal vibration numbers probably cannot be obtained. For low Péclet number, however, a perturbation about the no-flow or conduction solution might yield the desired results. Griffiths and Morrison [7] have shown that for the flow about a drop in a steady electric field, the conduction solution is a uniformly valid zeroth order approximation for undertaking a regular perturbation analysis in Péclet number. This, of course, is not the case for the flow about a translating body [13]. The arguments are unchanged for the flow generated by an alternating electric field and one would expect a regular expansion to be useful in solving this problem. Note that because the oscillating part of the fluid speed is never less than the steady part, the widely used Lighthill–Illingworth technique [14, 15] is not applicable.

We seek a solution to the governing equation (17) in the form

$$\mathrm{T} = \sum_{j=0}^{\infty}\sum_{k=0}^{\infty} \varepsilon^j \delta^k \mathrm{T}^{j,k}(R, Z, \tau; \delta), \tag{22}$$

$$\mathrm{T}^{j,k}(R, Z, \tau; \delta) = \sum_{m=-\infty}^{\infty}\sum_{p=-\infty}^{\infty}\sum_{n=0}^{\infty} G^{j,k}_{m,p,n}(R) \times e^{-C_p(R-1)/\delta}\, e^{im\tau} P_n(Z) \tag{23}$$

where

$$C_p = (ip)^{1/2} \quad \text{such that} \quad \mathrm{Re}[C_p] \geqslant 0. \tag{24}$$

$P_n(Z)$ is the nth order Legendre polynomial.

Several comments on the form of the proposed solution seem appropriate. First, the quotient

$$x \equiv (R-1)/\delta \tag{25}$$

is clearly an inner variable. It is zero on the drop surface and varies dramatically with small changes in the dimensionless radial position. The inner solutions ($p \neq 0$) involve not only the inner variable x, but the outer variable R as well. Owing to the exponential dependence on x, the inner solutions approach zero at a small distance from the drop surface. At large distance from the surface, behavior of the temperature field is dependent only on the outer solutions ($p = 0$) and the outer variable R. Thus, the form of the proposed solution lucidly displays an inner and an outer behavior. Second, if

$$\left.\begin{aligned} &\mathrm{T}^{j,k}(R, Z, \tau; \delta) = O[1] \\ &\text{then} \\ &\frac{\partial}{\partial R}[\mathrm{T}^{j,k}(R, Z, \tau; \delta)] = O[1/\delta] \end{aligned}\right\} \text{as } \delta \to 0 \tag{26}$$

in general. This disparity in the order of the temperature and its derivatives would preclude a regular perturbation solution. Finally, a comment on the types of time dependence the proposed solution may describe: the two exponentials in equation (23) will combine to yield

$$\sin \text{ or } \cos\,[m\tau \pm (|2p|)^{1/2}(R-1)/2\delta]. \tag{27}$$

The inner solutions, therefore, are periodic in time with a radially dependent phase shift. This is wave-like behavior and originates in the period generation of disturbances in the temperature field (by the periodic fluid motion) which then propagate toward or away from the drop. The outer solutions are likewise periodic but without the phase shift. Since we are concerned here only with the quasi-steady transport, the issue of early and late time behavior, i.e. small or large values of τ, does not arise. All time dependent terms in the solutions will be periodic, of one form or another.

To satisfy the boundary condition on the drop surface, equation (20), for all ε and δ we must have

$$T^{0,0}(R,Z,\tau;\delta) = 1 \quad \text{on} \quad R = 1 \tag{28}$$

and

$$T^{j,k}(R,Z,\tau;\delta) = 0 \quad \text{on} \quad R = 1 \tag{29}$$

for higher order approximations. In more detail, these conditions may be written as

$$\sum_{p=-\infty}^{\infty} G^{0,0}_{0,p,0}(R) = 1 \quad \text{on} \quad R = 1 \tag{30}$$

$$\sum_{p=-\infty}^{\infty} G^{0,0}_{m,p,n}(R) = 0 \quad \text{on} \quad R = 1 \quad m \text{ or } n \neq 0 \tag{31}$$

for equation (28), and

$$\sum_{p=-\infty}^{\infty} G^{j,k}_{m,p,n}(R) = 0 \quad \text{on} \quad R = 1 \quad j \text{ or } k \neq 0 \tag{32}$$

for equation (29). As will soon be evident, these conditions can be satisfied only by virtue of being summations. This is an essential feature of the proposed solution because the individual terms inside the summation of equations (30), (31), and (32) cannot, in general, be made to vanish on the drop surface.

Far drom the drop, the second boundary condition, equation (21), becomes

$$T^{j,k}(R,Z,\tau;\delta) \to 0 \quad \text{as} \quad R \to \infty \tag{33}$$

for all j and k. For the outer solutions, this requires that

$$G^{j,k}_{m,0,n}(R) \to 0 \quad \text{as} \quad R \to \infty \tag{34}$$

for all j, k, m, and n. For the inner solutions, it is necessary only that

$$G^{j,k}_{m,p,n}(R) = o[e^{(|2p|^{1/2})R/2\delta}] \quad \text{as} \quad R \to \infty \tag{35}$$

for all $j, k, m, p \neq 0$, and n. This is a very weak condition and may be replaced by the more restrictive relation

$$G^{j,k}_{m,p,n}(R) = O[R^b] \quad \text{as} \quad R \to \infty \tag{36}$$

for arbitrary b and all j, k, m, $p \neq 0$, and n.

Substituting equations (22) and (23) into the governing equation (17) and applying several identities for the Legendre polynomials gives, with minor rearrangement,

$$\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{m=-\infty}^{\infty}\sum_{p=-\infty}^{\infty}\sum_{n=0}^{\infty}\varepsilon^j\delta^k \times [\mathrm{i}(m-p)G^{j,k}_{m,p,n}(R)P_n(Z)\,e^{-C_p(R-1)/\delta}\,e^{\mathrm{i}m\tau}]$$

$$= \sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{m=-\infty}^{\infty}\sum_{p=-\infty}^{\infty}\sum_{n=0}^{\infty}\varepsilon^j\delta^k \times \Bigg\{\delta^2\bigg[R^{-2}\frac{\mathrm{d}}{\mathrm{d}R}\left(R^2\frac{\mathrm{d}}{\mathrm{d}R}G^{j,k}_{m,p,n}(R)\right) - R^{-2}n(n+1)G^{j,k}_{m,p,n}(R)\bigg] - 2\delta C_p R^{-1}\frac{\mathrm{d}}{\mathrm{d}R}\bigg[RG^{j,k}_{m,p,n}(R)\bigg] + \varepsilon\sum_{q=-1}^{1}\sum_{l=-2,0}^{2}[\eta_s + |q|(\tfrac{1}{2}\eta_t - \eta_s)]$$
$$\times\bigg[(R^{-4} - R^{-2})A_l(n-l) \times\left(-\delta^2\frac{\mathrm{d}}{\mathrm{d}R}G^{j,k}_{m-q,p,n-l}(R) + \delta C_p G^{j,k}_{m-q,p,n-l}(R)\right) + (2R^{-5})B_l(n-l)\left(-\delta^2 G^{j,k}_{m-q,p,n-l}(R)\right)\bigg]\Bigg\}$$
$$\times P_n(Z)\,e^{-C_p(R-1)/\delta}\,e^{\mathrm{i}m\tau} \tag{37}$$

where

$$A_{-2}(n) = 3n(n-1)/(2n-1)(2n+1),$$
$$A_0(n) = 2n(n+1)/(2n-1)(2n+3), \tag{38}$$
$$A_2(n) = 3(n+1)(n+2)/(2n+1)(2n+3)$$

and

$$B_{-2}(n) = -n(n-1)(n+1)/(2n-1)(2n+1),$$
$$B_0(n) = -n(n+1)/(2n-1)(2n+3), \tag{39}$$
$$B_2(n) = n(n+1)(n+2)/(2n+1)(2n+3).$$

When like powers of ε and δ are equated, equation (37) may be solved termwise. If we allow

$$G^{j,k}_{m,p,n}(R) = 0, \quad j \text{ or } k < 0 \tag{40}$$

then the zeroth and first order expressions need not be written separately; the following expression is valid for all j and k.

$$\mathrm{i}(m-p)G^{j,k}_{m,p,n}(R) = R^{-2}\frac{\mathrm{d}}{\mathrm{d}R}\left(R^2\frac{\mathrm{d}}{\mathrm{d}R}G^{j,k-2}_{m,p,n}(R)\right) - R^{-2}n(n+1)G^{j,k-2}_{m,p,n}(R) - 2C_pR^{-1}\frac{\mathrm{d}}{\mathrm{d}R}\left(RG^{j,k-1}_{m,p,n}(R)\right)$$
$$+ \sum_{q=-1}^{1}\sum_{l=-2,0}^{2}[\eta_s + |q|(\tfrac{1}{2}\eta_t - \eta_s)] \times\bigg[(R^{-4} - R^{-2})A_l(n-l) \times\left(-\frac{\mathrm{d}}{\mathrm{d}R}G^{j-1,k-2}_{m-q,p,n-l}(R) + C_pG^{j-1,k-1}_{m-q,p,n-l}(R)\right)$$
$$+ (2R^{-5})B_l(n-l)\left(-G^{j-1,k-2}_{m-q,p,n-l}(R)\right)\bigg]. \tag{41}$$

Observe that equation (41) is, in general, a zeroth order differential equation. It possesses no non-trivial homogeneous solutions and for this reason the particular solutions cannot be modified to satisfy any boundary conditions. There are, however, two special cases that need be considered.

When m and p are equal but not zero, the LHS of equation (41) vanishes. This first degenerative case is handled by moving the third term on the RHS to the LHS. The new expression must still be valid for all k greater than or equal to zero. By equation (40), however, the first result is trivial ($0 = 0$). Replacing k by $k+1$ (i.e. incrementing the arbitrary k index) gives

$$2C_m R^{-1}\frac{\mathrm{d}}{\mathrm{d}R}\left(RG^{j,k}_{m,m,n}(R)\right) = R^{-2}\frac{\mathrm{d}}{\mathrm{d}R}\left(R^2\frac{\mathrm{d}}{\mathrm{d}R}G^{j,k-1}_{m,m,n}(R)\right) - n(n+1)R^{-2}G^{j,k-1}_{m,m,n}(R) + \sum_{q=-1}^{1}\sum_{l=-2,0}^{2} \times[\eta_s+|q|(\tfrac{1}{2}\eta_t-\eta_s)] \times\left[(R^{-4}-R^{-2})A_l(n-l) \times\left(-\frac{\mathrm{d}}{\mathrm{d}R}G^{j-1,k-1}_{m-q,m,n-l}(R)+C_m G^{j-1,k}_{m-q,m,n-l}(R)\right) + (2R^{-5})B_l(n-l)\left(-G^{j-1,k-1}_{m-q,m,n-l}(R)\right)\right]. \tag{42}$$

Equation (42) is a first-order differential equation with the single homogeneous solution

$$H^{j,k}_{m,m,n}(R) = R^{-1}, \quad m \neq 0. \tag{43}$$

The particular solution can be obtained directly by integration. Combining the particular solution with a constant multiple of the homogeneous solution allows the boundary condition on the drop surface, equation (31) or (32), to be satisfied for the terms of all degrees in m other than zero.

In the second degenerative case, when m and p are equal and zero, the LHS of equation (42) additionally vanishes. Again the equation may be rearranged; this time the first two terms on the RHS are brought to the LHS. A second increment in the k index gives

$$-R^{-2}\frac{\mathrm{d}}{\mathrm{d}R}\left(R^2\frac{\mathrm{d}}{\mathrm{d}R}G^{j,k}_{0,0,n}(R)\right)+n(n+1)R^{-2}G^{j,k}_{0,0,n}(R) = \sum_{q=-1}^{1}\sum_{l=-2,0}^{2}[\eta_s+|q|(\tfrac{1}{2}\eta_t-\eta_s)] \times\left[(R^{-4}-R^{-2})A_l(n-l)\left(-\frac{\mathrm{d}}{\mathrm{d}R}G^{j-1,k}_{-q,0,n-l}(R)\right) + (2R^{-5})B_l(n-l)\left(-G^{j-1,k}_{-q,0,n-l}(R)\right)\right]. \tag{44}$$

This is an Euler equation with homogeneous solutions

$$H^{j,k}_{0,0,n}(R) = R^{-(n+1)} \tag{45}$$

and

$$H^{j,k}_{0,0,n}(R) = R^{n}. \tag{46}$$

The particular solution may be obtained by use of the homogeneous solutions and the method of variation of parameters. The first homogeneous solution satisfies equation (34) and is therefore suitable for use in satisfying the boundary condition on the drop surface, equation (30) or (31), for all terms of degree zero in m.

THE ZERO–ZERO APPROXIMATION

The zero–zero approximation to the temperature field is governed by equations (41), (42), and (44) with both j and k equal to zero. Noting equation (40), these become

$$G^{0,0}_{m,p,n}(R) = 0, \quad m \neq p, \tag{47}$$

$$2C_m R^{-1}\frac{\mathrm{d}}{\mathrm{d}R}\left(RG^{0,0}_{m,m,n}(R)\right) = 0, \quad m \neq 0, \tag{48}$$

$$-R^{-2}\frac{\mathrm{d}}{\mathrm{d}R}\left(R^2\frac{\mathrm{d}}{\mathrm{d}R}G^{0,0}_{0,0,n}(R)\right) + n(n+1)R^{-2}G^{0,0}_{0,0,n}(R) = 0. \tag{49}$$

The solutions to the equations are simply the homogeneous solutions, equations (43) and (45). The boundary conditions may, therefore, be written

$$\sum_{p=-\infty}^{\infty} D_{p,n}H^{0,0}_{0,p,n}(R) = D_{0,n}H^{0,0}_{0,0,n}(R) = \begin{vmatrix} 1 & n=0, \\ 0 & n\neq 0 \end{vmatrix} \tag{50}$$

where $D_{p,n}$ are constants. Also,

$$\sum_{p=-\infty}^{\infty} D_{p,n}H^{0,0}_{m,p,n}(R) = D_{m,n}H^{0,0}_{m,m,n}(R) = 0. \tag{51}$$

The values satisfying these relations are

$$D_{0,0} = 1, \quad D_{m,n} = 0, \quad m \text{ or } n \neq 0. \tag{52}$$

The familiar result is

$$G^{0,0}_{0,0,0}(R) = 1/R, \tag{53}$$

$$G^{0,0}_{m,p,n}(R) = 0, \quad m, p, \text{ or } n \neq 0. \tag{54}$$

This is the conduction or no-flow approximation, as would be expected.

ZERO–k APPROXIMATIONS

The governing equations for the zero-k approximations are given by

$$\mathrm{i}(m-p)G^{0,k}_{m,p,n}(R) = R^{-2}\frac{\mathrm{d}}{\mathrm{d}R}\left(R^2\frac{\mathrm{d}}{\mathrm{d}R}G^{0,k-2}_{m,p,n}(R)\right) - n(n+1)R^{-2}G^{0,k-2}_{m,p,n}(R) - 2C_pR^{-1}\frac{\mathrm{d}}{\mathrm{d}R}\left(RG^{0,k-1}_{m,p,n}(R)\right), \quad m \neq p, \tag{55}$$

$$2C_mR^{-1}\frac{d}{dR}\left(RG^{0,k}_{m,m,n}(R)\right) = R^{-2}\frac{d}{dR}\left(R^2\frac{d}{dR}G^{0,k-1}_{m,m,n}(R)\right) - n(n+1)R^{-2}G^{0,k-1}_{m,m,n}(R), \quad m \neq 0. \tag{56}$$

$$-R^{-2}\frac{d}{dR}\left(R^2\frac{d}{dR}G^{0,k}_{0,0,n}(R)\right) + n(n+1)R^{-2}G^{0,k}_{0,0,n}(R) = 0. \tag{57}$$

Recall that the zero–zero approximation is non-trivial only for m and p equal to zero. It is evident from this that the zero–one solutions which satisfy the boundary condition on the drop surface will be only the trivial solutions. The argument can be applied repeatedly to conclude that

$$G^{0,k}_{m,p,n}(R) = 0, \quad k \geqslant 1 \tag{58}$$

for all m, p, and n. This result seems reasonable as the zero–k approximations are, again, no-flow approximations. In the absence of time dependent fluid motion, the quasi-steady temperature field is the steady field and as such would be independent of the thermal vibration number—regardless of its magnitude.

j ZERO APPROXIMATIONS

The last of the zeroth order approximations, the j-zero approximations, are governed by

$$G^{j,0}_{m,p,n}(R) = 0, \quad m \neq p, \tag{59}$$

$$2C_mR^{-1}\frac{d}{dR}\left[RG^{j,0}_{m,m,n}(R)\right] = \sum_{q=-1}^{1}\sum_{l=-2,0}^{2}[\eta_s + |q|(\tfrac{1}{2}\eta_t - \eta_s)] \times (R^{-4} - R^{-2})A_l(n-l)C_mG^{j-1,0}_{m-q,m,n-l}(R), \quad m \neq 0, \tag{60}$$

$$R^{-2}\frac{d}{dR}\left(R^2\frac{d}{dR}G^{j,0}_{0,0,n}(R)\right) + n(n+1)R^{-2}G^{j,0}_{0,0,n}(R) = \sum_{q=-1}^{1}\sum_{l=-2,0}^{2}[\eta_s + |q|(\tfrac{1}{2}\eta_t - \eta_s)]\Big[(R^{-4} - R^{-2})A_l(n-l) \times \left(-\frac{d}{dR}G^{j-1,0}_{q,0,n-l}(R)\right) + (2R^{-5})B_l(n-l) \times (\Psi - G^{j-1,0}_{-q,0,n-l}(R))\Big]. \tag{61}$$

Equation (59) shows that non-trivial solutions will occur only for m equal to p. Again recalling that the zero–zero approximation is non-trivial only for m and p equal to zero, it becomes evident from equations (60) and (61) and the necessary boundary conditions that the non-trivial one–zero solutions will also occur only for m and p equal to zero. This reasoning can be applied repeatedly to obtain a single expression for all non-trivial j zero solutions.

$$-R^{-2}\frac{d}{dR}\left(R^2\frac{d}{dR}G^{j,0}_{0,0,n}(R)\right) + n(n+1)R^{-2}G^{j,0}_{0,0,n}(R) = \sum_{l=-2,0}^{2}\eta_s\Big[(R^{-4} - R^{-2})A_l(n-l)\left(-\frac{d}{dR}G^{j-1,0}_{0,0,n-l}(R)\right) + (2R^{-5})B_l(n-l)\left(-G^{j-1,0}_{0,0,n-l}(R)\right)\Big]. \tag{62}$$

The remaining solutions are

$$G^{j,0}_{m,p,n}(R) = 0, \quad m \text{ or } p \neq 0. \tag{63}$$

The non-trivial j zero approximations may also be interpreted through physical arguments. As the thermal vibration number increases without bound ($\delta \to 0$), fluctuations in the fluid motion take place on a time scale much smaller than the thermal relaxation time. The temperature field cannot respond to such fluctuations and so is influenced only by conduction and the convective transport due to the steady part of the fluid motion.

In effect, equation (62) with the zero zero approximation and appropriate boundary conditions previously has been solved by Griffiths and Morrison [7]. Simply replacing ε in their solutions by $\varepsilon\eta_s$ yields the correct results for the j-zero approximations. This fact will serve as a significant check on later computations.

HIGHER ORDER APPROXIMATIONS

Before attempting to compute any higher order approximations, an interest in efficiency requires bounding the solution subscripts, i.e. requires determining an upper bound (UB) on the absolute value of the three subscripts m, p, and n for which non-trivial solutions may occur. This is accomplished by careful scrutiny of equations (41), (42), and (44), and the zeroth order approximations. The perspicacious reader can derive the following results with relative ease. If a relation does not involve either j or k, then the bound is valid for all values of that superscript.

$$\text{UB}[n] = 2j. \tag{64}$$

In addition, non-trivial solutions occur only for even values of n. This could have been noted *a priori* by observing that the fluid velocity, equations (2) and (3), is symmetric with respect to both the polar axis and equatorial plane of the drop. Also

$$\text{UB}[m] = \min(j, k)$$

and

$$\text{UB}[p] = \tfrac{1}{2}[m + \min(j, k)]. \tag{65}$$

Note, after the fact, that the zero k and j zero approximations satisfy these constraints.

Further consideration of the governing equations shows that

$$G^{j,1}_{m,p,n}(R) = 0 \quad \text{and} \quad G^{j,3}_{m,p,n}(R) = 0. \tag{66}$$

The first of these results follows immediately from the zero-k and j-zero approximations being limited to degree zero in m and p and the need to satisfy the boundary condition on the drop surface. Trial substitution of the zeroth order approximations into the j one governing equations will readily show that this is correct. The second follows from similar, although more complex, arguments. These two results will provide a valuable check on the computation of higher order approximations.

Computation by hand yields the next few approximations to the temperature distribution using the same methods used to obtain the zeroth order solutions. To first order in ε and fourth order in δ, the non-zero results are

$$\begin{aligned}
G^{1,0}_{0,0,2}(R) &= \eta_s(-\tfrac{1}{3}R^{-4}+\tfrac{5}{6}R^{-3}-\tfrac{1}{2}R^{-2}),\\
G^{1,2}_{-1,0,2}(R) &= \eta_t(\mathrm{i}R^{-6}-\mathrm{i}R^{-4}),\\
G^{1,2}_{1,0,2}(R) &= -\eta_t(\mathrm{i}R^{-6}-\mathrm{i}R^{-4}),\\
G^{1,4}_{-1,0,2}(R) &= -\eta_t(24R^{-8}-6R^{-6}),\\
G^{1,4}_{1,0,2}(R) &= -\eta_t(24R^{-8}-6R^{-6}),\\
G^{1,4}_{1,1,2}(R) &= \eta_t(18R^{-1}),\\
G^{1,4}_{1,-1,2}(R) &= \eta_t(18R^{-1}).
\end{aligned} \tag{67}$$

HEAT TRANSFER

Corresponding to the form of equation (22), we seek an expression for the overall time-averaged heat transfer coefficient, expressed as the Nusselt number, in the form

$$\overline{Nu} = \sum_{j=0}^{\infty}\sum_{k=0}^{\infty} \varepsilon^j\delta^k\bar{Q}^{j,k}(\delta) = \sum_{j=0}^{\infty}\sum_{k=0}^{\infty} \varepsilon^j\delta^k\overline{Nu}^{j,k}. \tag{68}$$

Nu is the overall Nusselt number and an overbar denotes averaging with respect to time. The zeroth and all higher order contributions are found by integrating the normal flux over the drop surface

$$Q^{j,k}(\delta) = -\int_{-1}^{1} \frac{\partial}{\partial R}T^{j,k}(R,Z,\tau;\delta)\bigg|_{R=1}\,\mathrm{d}Z. \tag{69}$$

The Legendre polynomials are, however, an orthogonal set over the interval negative one to one with a weight function of unity. Since $P_0(Z)$ is by definition unity, only terms of degree zero in n need be considered. Additionally,

$$\overline{e^{\mathrm{i}m\tau}} = 0, \quad m \neq 0 \tag{70}$$

so we need also consider only the solutions of degree zero in m. Thus, the time average at equation (69) gives

$$\overline{Nu}^{j,k} = -2\sum_{p=-\infty}^{\infty}\left[\frac{\mathrm{d}}{\mathrm{d}R}G^{j,k}_{0,p,0}(R) - C_p G^{j,k+1}_{0,p,0}(R)\right]\bigg|_{R=1}. \tag{71}$$

This unusual form follows directly from equation (26).

Using equation (71) and the approximations previously obtained, we find that

$$\overline{Nu}^{0,0} = 2, \tag{72}$$

$$\overline{Nu}^{0,k} = 0, \quad k = 1,2,3, \tag{73}$$

$$\overline{Nu}^{1,k} = 0, \quad k = 0,1,2,3. \tag{74}$$

The first of these is, of course, the well known result for transfer due to conduction. The second contribution is in agreement with the results for low Péclet number transport in a steady field [7].

To the order obtained so far, the heat transfer coefficient is independent of the thermal vibration number. One may reasonably entertain the notion that the coefficient is independent of this parameter at all orders of approximation, i.e. that the periodic part of the fluid motion contributes nothing to the time-averaged transport. The notion is dismissed, however, by observing that the limits of very large and very small thermal vibration number do not yield the same temperature distribution or heat transfer coefficient.

Consider, for a moment, the limit of very large thermal vibration number ($\delta \to 0$). In this case the solutions for the temperature field are the j-zero solutions previously discussed and governed by equation (62). The temperature distribution is that due only to the steady part of the fluid motion. Because the solutions contain no time dependence, the instantaneous and time-averaged transfer coefficients are the same. As mentioned, this problem has, in effect, been solved. The exercise indicated gives

$$\overline{Nu} = 2+\left[\frac{41}{3150}\eta_s^2\right]\varepsilon^2+O[\varepsilon^4] \quad \text{as} \quad \delta \to 0 \tag{75}$$

for a very large thermal vibration number.

Now, consider the limit of very small thermal vibration number ($\delta \to \infty$). In this limit, oscillations in the fluid motion take place on a time scale much larger than the thermal relaxation time. The time dependent temperature distribution is, therefore, that due to the instantaneous fluid velocity. The solutions are the same as those for the small limit except that $\varepsilon\eta_s$ is replaced by $\varepsilon[\eta_s+\eta_t\cos(\tau)]$. A similar substitution in equation (75) gives the instantaneous heat transfer coefficient. The time-averaged coefficient is

$$\overline{Nu} = 2+\left[\frac{41}{3150}(\eta_s^2+\tfrac{1}{2}\eta_t^2)\right]\varepsilon^2+O[\varepsilon^4] \quad \text{as} \quad \delta \to \infty \tag{76}$$

for very small thermal vibration number.

Thus, the heat transfer coefficients in the limits of large and small thermal vibration number are not the same except for η_t identically zero, i.e. for no time dependent fluid motion. The transport in an alternating field cannot be independent of δ and so at least one non-zero contribution to the heat transfer coefficient must exist for k not equal to zero.

Clearly, to obtain non-vanishing contributions to the heat transfer coefficient requires an unusually high order approximation to the temperature distribution. Beyond the results obtained so far, these approximations quickly become unreasonable to compute by hand; the size of the solutions grow very rapidly with increasing order. One also faces the high probability of human error in computing higher order approximations. To overcome these difficulties, we sought a computer solution. The result is a PL/I FORMAC [17–19] implemented program which analytically solves the governing equations, satisfies the boundary conditions, and computes contributions to the heat transfer coefficient. The program uses precisely the same algorithm described earlier for computation by hand.

To date, the full approximations to the temperature distribution have been computed to second order in ε and fourth order in δ. Beyond this level, contributions to the transfer coefficient have been obtained without computing all of the temperature solutions. This is achieved through a technique dubbed pruning.

Because the contribution to heat transfer comes from only a part of any approximation to the temperature field, only certain of the temperature solutions need be computed; recall equation (71), only the solutions for which m and n are both zero are involved. At any level of approximation, however, all solutions that influence desired higher order solutions for which m and n are zero must also be computed. This can be expressed simply by writing new bounds on the solution subscripts.

$$\mathrm{UB}[n] = \min(2j, 2[j_{\max} - j]) \tag{77}$$

and

$$\mathrm{UB}[m] = \min(j, j_{\max} - j, k) \tag{78}$$

where $j_{\max}$ is the maximum order desired in ε. The bound on subscript p cannot be improved. By equation (65), however, the range of p will be reduced by the new bound on m. Observe that up to order $j_{\max}/2$ in ε, the new bounds are the same as those given previously; all solutions to this order need be computed. Beyond this order, progressively fewer solutions are computed; those which are extraneous to higher order contributions to the heat transfer coefficient are pruned from the solution set.

The contributions to the heat transfer coefficient have been computed to fourth order in ε and eighth order in δ. To the lowest non-zero contribution, the result is

$$\dot{N}u = 2+\varepsilon^2\left[\frac{41}{3150}\eta_s^2+\left(\frac{272}{1155}\eta_t^2\right)\delta^4+O(\delta^7)\right] - \varepsilon^4\eta_s^2\left[\frac{66245909}{1376633758500}\eta_s^2 + \left(\frac{57174512}{57489807375}\eta_t^2\right)\delta^4+O(\delta^7)\right]. \tag{79}$$

Observe that

$$\ddot{N}u^{j,k} = 0, \quad j = 1, 3 \tag{80}$$

for all orders in k. This result is probably not a mere characteristic of the low order approximations. Brenner [20] has shown that the overall steady rate of heat transfer from an isothermal body to a fluid at uniform temperature is left unchanged when the fluid velocity is reversed at every point. Morrison and Griffiths [21] have shown that this result holds for the transient transfer rate. We might expect that the result would also hold for the present problem. Noting that ε in equation (17) may be either positive or negative, it would then follow that the first and third order result, equation (80), should hold for all approximations of odd order in j.

Because a high order approximation to the heat transfer coefficient has been computed, we are in a position to estimate [22, 23] the radii of convergence of the series. By comparing the computed coefficients, we can estimate that the series will converge for

$$|\varepsilon| < 15 \quad \text{and} \quad \delta < 0.97 \ (\zeta > 200). \tag{81}$$

Beyond these values, the terms in the summations of the Nusselt number become about equal in magnitude and the series would be expected to diverge. In practice, however, the useful range of the series may extend well above these values owing to the alternating signs of higher order coefficients.

Finally, we consider the high Péclet number heat transfer in an alternating electric field. With relative ease, the behavior in the limits of very large and very small thermal vibration number may be obtained. The physical arguments given for low Péclet number transport in these two limits apply equally to the high Péclet number domain. In this instance, the analogous transport in a steady electric field has been treated by Morrison [8]. Replacing Pe in his analysis by $\eta_s Pe$ or $[\eta_s + \eta_t \cos(\tau)]Pe$ gives the desired results. A special case of his high Péclet number analysis is the steady behavior for dominant thermal resistance in the surrounding fluid. For very large thermal vibration number, the instantaneous and time-averaged transfer coefficients are

$$Nu = \bar{N}u = 2\eta_s^{1/2}(Pe/\pi)^{1/2} \quad \text{as} \quad \delta \to 0. \tag{82}$$

Also,

$$Nu = 2f(\tau, \eta_t/\eta_s)\eta_s^{1/2}(Pe/\pi)^{1/2} \quad \text{as} \quad \delta \to \infty, \tag{83}$$

$$f(\tau, \eta_t/\eta_s) = [1+(\eta_t/\eta_s)\cos(\tau)]^{1/2} \tag{84}$$

for very small thermal vibration number.

The time-average of equation (84), and thus of equation (83), has been obtained numerically. We find that

$$\bar{N}u(Pe;\delta\to\infty)/\bar{N}u(Pe;\delta\to 0) = \bar{f}(\eta_t/\eta_s)$$

$$\begin{aligned} &= 0.90 & & &= 1.00 \\ &< 1.00 & & &< 1.54 \\ & &\text{for } \eta_t/\eta_s & & \\ &> 1.00 & & &> 1.54 \\ &\sim 0.76(\eta_t/\eta_s)^{1/2} & & &\to \infty \end{aligned} \tag{85}$$

for very large Péclet number.

CONCLUSIONS

The fundamental parameters associated with quasi-steady heat or mass transfer from a drop in an alternating electric field have been identified as the Péclet number and square-root of the thermal vibration number. A solution to the energy transport equation for low Péclet number and high thermal vibration number was found in the form of a composite, double perturbation expansion. As in the analysis of transport in a steady electric field, the expansion is regular in Péclet number. A digital computer was used to obtain high order solutions to the recursive governing equations.

The overall time-averaged heat transfer coefficient, expressed as the Nusselt number, was found to be a weak function of the thermal vibration number; above 200, transport is almost entirely due to the steady part of the creeping fluid motion. In contrast, for very small thermal vibration number, the steady and the oscillating parts of the fluid motion make nearly equal contributions to overall heat or mass transfer. For low Péclet number, the transfer rate is always higher for very low thermal vibration number than for very high thermal vibration number. The periodic part of the fluid motion tends to enhance the transfer rate.

The limits of very low and very high thermal vibration number have also been considered for high Péclet number transport. Unlike the low Péclet number case, the periodic part of the fluid motion may either enhance or detract from the overall time-averaged heat transfer rate.

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LE TRANSFERT A PARTIR D'UNE GOUTTE DANS UN CHAMP ELECTRIQUE ALTERNATIF

Résumé – Les analyses du transfert de chaleur et de masse à partir d'une goutte dans un champ électrique ont jusqu'à présente porté sur des champs électriques stationnaires. Cette étude présente à la fois des solutions de nombre de Péclet élevé et faible pour un champ alternatif. Le transfert à nombre de Péclet faible est étudié analytiquement par un développement à perturbation double et composite. Des méthodes analytiques spéciales sont développées pour considérer le problème jamais traité où les composantes permanentes et celles dépendantes du temps pour le mouvement du fluide sont du même ordre de grandeur. Un calculateur digital est utilisé pour obtenir des solutions exactes des équations récurrentes. Ces solutions donnent des résultats précis pour un nombre de Péclet entre zéro et trente et des nombres de vibration thermique jusqu'à deux cents.

DIE TRANSPORTVORGÄNGE AN EINEM TROPFEN IN EINEM ELEKTRISCHEN WECHSELFELD

Zusammenfassung Die Untersuchungen des Wärme- und Stoffübergangs an einem Tropfen in einem elektrischen Feld wurden bis heute nur an konstanten elektrischen Feldern durchgeführt. Die vorliegende Untersuchung liefert Lösungen bei sowohl hohen als auch niedrigen Peclet-Zahlen für den Transport in einem elektrischen Wechselfeld. Der Transport bei niedrigen Peclet-Zahlen wurde analytisch mit einem Störungsansatz und einer zusammengesetzten doppelten Reihenentwicklung untersucht. Spezielle analytische Verfahren wurden entwickelt, um das bisher unbehandelte Transportproblem zu untersuchen, wobei die stationären und zeitabhängigen Anteile der Fluidbewegung von gleicher Größenordnung sind. Ein Digitalrechner wurde eingesetzt, um exakte Lösungen der rekursiven Bilanzgleichungen zu erhalten. Diese Lösungen liefern genaue Ergebnisse im Bereich der Peclet-Zahlen von null bis dreißig und einer thermischen Schwingungszahl von über zweihundert.

ПРОЦЕССЫ ПЕРЕНОСА ОТ КАПЛИ В ПЕРЕМЕННОМ ЭЛЕКТРИЧЕСКОМ ПОЛЕ

Аннотация До настоящего времени анализ тепло-и массопереноса от капли, находящейся в электрическом поле, ограничивался только постоянными электрическими полями. В данной работе представлены решения для случая переноса в переменном электрическом поле как при большом, так и малом значениях числа Пекле. Перенос при малом значении числа Пекле исследовался аналитически с помощью сложного двукратного разложения в ряд по возмущениям. Разработаны специальные аналитические методы для исследования ранее не рассматривавшейся задачи переноса, в которой стационарные и зависящие от времени компоненты движения жидкости равны по величине. Точные решения основных рекуррентных уравнений были получены с помощью ЭВМ. Они позволяют получить надежные данные для диапазона значений числа Пекле от нуля до тридцати при частоте тепловых колебаний, превышающей 200.